PIPING STRESSES ■ STEAM TURBINE EFFICIENCY ■ TRAINING THE PLANT MANAGER

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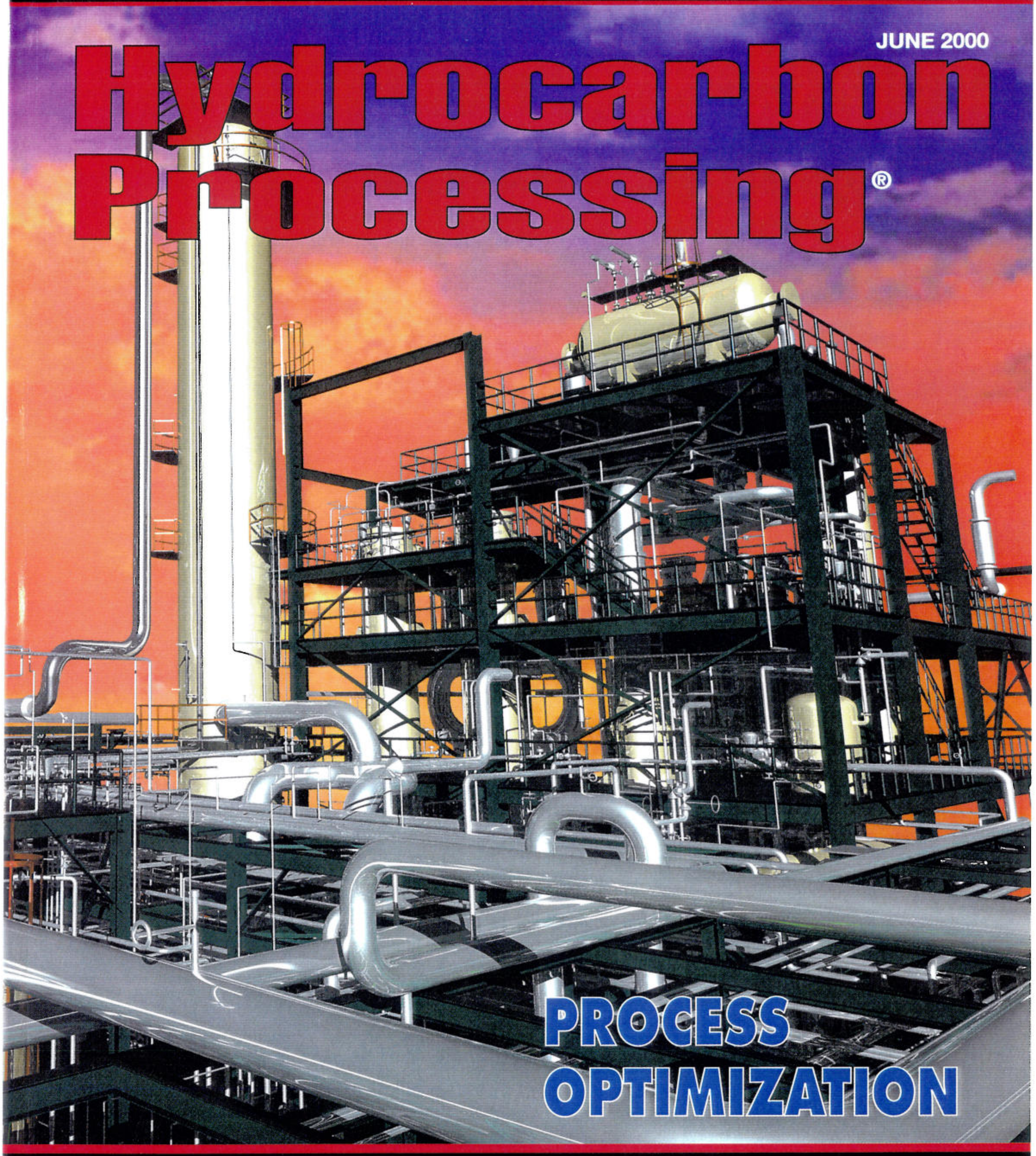

PROCESS OPTIMIZATION

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Evaluating piping dynamic stresses

Use these calculations for a quick, accurate analysis

K. T. Truong, The Ultragen Group Ltd., Quebec, Canada

Piping vibration can be a concern to utility owners and operators as well as regulatory agencies. Many programs have been developed to assure reliability and plant safety with respect to vibration; all are also aimed at minimizing cost and delay during plant startup. Acceptability of piping system vibration is affected by the maximum vibratory stress in the pipe. This could be determined by either visual observation or, obviously, by more elaborate instrumentation-based measurement and analytical techniques.

In either case, the dynamic stress should not exceed an allowable level related to permissible alternating stress values given by the ASME Code for a given number of cycles. Since direct dynamic stress measurement is a complicated process, vibration is mainly monitored by using portable instruments that capture frequency and amplitude.

The following approach presents a rapid and reliable means of evaluating the harmonic dynamic stresses of a simply supported pipeline from data collected in the field. Moreover, it illustrates how a calculator or personal computer might be all that's needed to determine dynamic stresses in piping. This may come in handy if the analyst is expected to provide answers without being able to resort to computer software.

After explaining the calculation basis and underlying equations, we will highlight the simplicity and relative accuracy of the "emergency calculator approach" in a case history.

Basic equations. Free vibration occurs when a system is displaced from its static position and left free to oscillate. Under free vibration the system oscillates at its *natural frequencies*. The natural frequencies are dynamic characteristics of the system specified by its stiffness and inertia properties. Natural frequencies are calculated with *modal analysis*. Forced vibrations are classified as either *periodic* or *nonperiodic*. In periodic vibration, the response repeats itself at a regular time interval, called *period T*. Harmonic excitation is a sub-class of periodic vibration and is described in this article as an *analytical approach*.

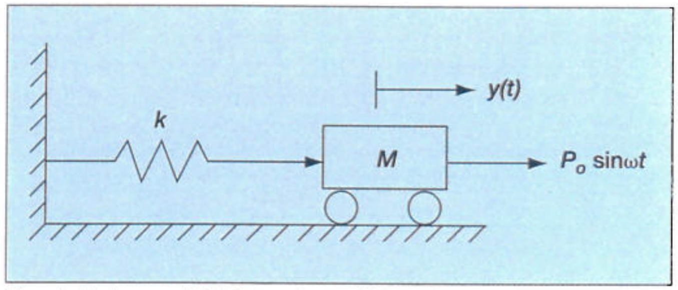

Fig. 1. Undamped SDOF system subjected to a harmonic load.

Consider an undamped single degree of freedom (SDOF) system that is subjected to a harmonic force, $P(t)$, with amplitude P_0 and circular frequency $\overline{\omega}$ (Fig. 1). The equation of motion is given by:

$$M\ddot{y} + ky = P_0 \sin(\overline{\omega}t) \tag{1}$$

The solution of Eq. 1 is:

$$y(t) = A\cos\omega t + B\sin\omega t + \frac{P_0}{k}\frac{1}{1-r^2}\sin\overline{\omega}t \tag{2}$$

where r is defined as the ratio of the circular frequency of the externally applied load to the natural circular frequency of the system, that is:

$$r = \frac{\overline{\omega}}{\omega} = \frac{\bar{f}}{f} \tag{3}$$

The solution given by Eq. 2 is the superposition of the free vibration problem and the effect of the exciting force exposed by the last term of Eq. 2, which involves only the harmonic load frequency. For frequency response analysis or steady-state harmonic analysis, only the steady-state response is considered, and Eq. 2 becomes:

$$y(t) = \frac{P_0}{k}\frac{1}{1-r^2}\sin\overline{\omega}t \tag{4}$$

The solution for maximum displacement from an unphased harmonic analysis is then:

$$\delta_{dyn} = \frac{P_0}{k}\frac{1}{\left(1-r^2\right)} \tag{5}$$

$$\text{Let } \frac{P_0}{k} = \delta_{static} \Rightarrow \delta_{yn} = \delta_{static}\Big/\left(1-r^2\right) \tag{6}$$

where k is the piping structural stiffness and δ_{static} is the static deflection of the system.

For the simple hinged support pipe of mass intensity, m, we introduce the boundary conditions:

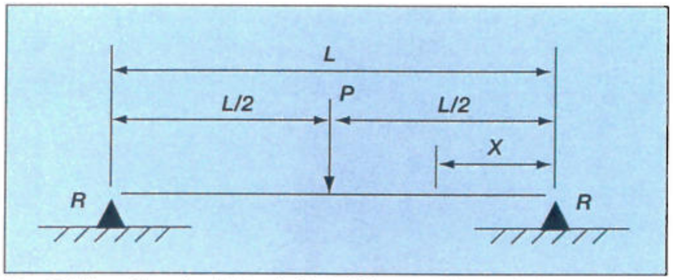

Fig. 2. When the pipe is loaded at mid-span, the static beam deflection equations are well known.

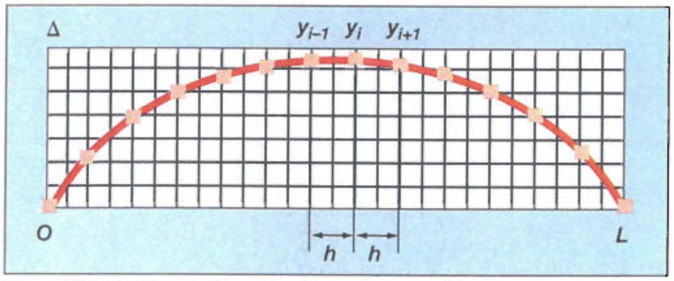

Fig. 3. Typical deflection of a hinged support beam.

$$y=0 \quad \text{at} \quad x=0,L \tag{B.C.1}$$

$$\text{and } \frac{d^2y}{dx^2}=0 \quad \text{at} \quad x=0,L \tag{B.C.2}$$

and the natural frequencies[1] are:

$$f_n = \frac{n^2\pi}{2L^2}\sqrt{\frac{EI}{m}} \tag{7}$$

where E = Young's modulus, psi
I = Pipe cross section moment of inertia, in.^{4}
m = Mass intensity, lbs-sec^2/in.^{2}
L = Pipe length, in.
n = 1, 2, 3, . . .

When the pipe is loaded with a force F at mid-span, the static beam deflection equations (Fig. 2) are well known and written as[2]:

$$\text{At load, } \Delta = \frac{PL^3}{48EI} \tag{8}$$

$$\text{When } x<\frac{L}{2}\ \Delta = \frac{Px}{48EI}\left(3L^2-4x^2\right) \tag{9}$$

Deflection is symmetrical on both sides of the concentrated load position and the dynamic deflection could be evaluated using Eqs. 6, 8 and 9.

With a typical dynamic deflection profile as shown in Fig. 3, to determine dynamic stresses one needs only apply the well-known relationships:

$$M=-EI\frac{d^2y}{dx^2} \tag{10}$$

$$\text{and } \sigma = \frac{MD}{2I} \tag{11}$$

for bending moment and bending stresses.

The second derivative of Eq. 10 could be solved using the finite differences technique. Since we are interested in the maximum stress evaluation, the second derivative at mid-span is calculated as:

$$y_i'' = \frac{y_{i-1}-2y_i+y_{i+1}}{h^2} \tag{12}$$

where h = Equispaced argument
y_i = Dynamic deflection at load ($x = L/2$)
$y_{i+1} = y_{i-1}$ = Dynamic deflection at ($x = L/2 \pm h$)

The dynamic peak stress is then, using Eqs. 8, 9 and 12:

$$\sigma_{dyn} = \frac{P\,D}{48Ih^2\left(1-r^2\right)}\left(3L^2x-4x^3-L^3\right) \tag{13}$$

$$\text{and } \left(\frac{\delta}{\sigma}\right)_{dyn} = \frac{L^3h^2}{E\,D\left(3L^2x-4x^3-L^3\right)} \tag{14}$$

Observing relation 14, it is interesting to note that the ratio of dynamic deflection to dynamic stress is constant for a pipe size in consideration, and it is independent of the pipe wall thickness as well as the harmonic excitation frequency and load.

The same technique could be used to develop a general case where the concentrated load is at any point between the two supports (Fig. 4). The beam static deflection equations are:

$$\text{At load, } \Delta = \frac{Pa^2b^2}{3EIL} \tag{15}$$

$$\text{When } x<a \quad \Delta = \frac{Pbx}{6EIL}\left(L^2-b^2-x^2\right) \tag{16}$$

$$\text{When } x>a \quad \Delta = \frac{Paz}{6EIL}\left(L^2-a^2-z^2\right) \tag{17}$$

The moment due to a harmonic force excitation is then:

$$M = EI\left(\frac{y_{i-1}-2y_i+y_{i+1}}{h^2}\right) \tag{18}$$

where the term in brackets could either be measured directly on site, or calculated using Eq. 6 combined with Eqs. 15, 16 and 17.

Case study. Our case study shows the application of this mathematical development and compares the results with those produced by typical piping stress software that has dynamic analysis features incorporated.

The piping model shown in Fig. 5 has the following characteristics:

Total length	240 in.
Outside diameter	4.5 in.
Wall thickness	0.337 in.
Modulus of elasticity	30E6 psi
Mass density	0.2825 lb/in.^{3}
Number of elements	22
Boundary conditions	Simply supported.

A harmonic force of 200 lb is applied at its mid-span with an excitation frequency of 5Hz. In reality, these loads could perhaps be an inline pump with its own dead weight, bolted into the piping structure and running at a certain rpm. Let's evaluate the maximum dynamic stress and peak displacement.

In situ measurement and evaluation:

- Use a vibration probe and measure peak displacement at the center of the pipe span to get y_i and at an adjacent location of distance h from the center for y_{i-1} or y_{i+1}. The distance h is usually selected to be 4 or 5 times the nominal pipe size.
- Use Eq. 12 to calculate y_i''
- Use Eq. 10 to calculate the moment M=EIy_i''

- Use Eq. 11 to evaluate the peak dynamic stress.

Analytical calculation:

1. Fundamental frequency:

$$\text{Eq. } 7 \Rightarrow f = \frac{\pi}{2L^2}\sqrt{\frac{EI}{m}}$$

Moment of inertia:

$$I = 0.049087\left(D^4 - d^4\right)$$

$$= 0.049087\left(4.5^4 - 3.826^4\right)$$

$$= 9.61 \text{ in.}^4$$

Mass intensity :

$$m = \frac{\rho A}{g}$$

$$= 0.2825\left(\frac{1}{32.174}\right)\left[\frac{\pi}{4}\left(D^2 - d^4\right)\right]\left(\frac{1}{12}\right)$$

$$= 0.003225 \quad \text{lb-sec}^2/\text{in.}^2$$

Fundamental frequency :

$$f = \frac{\pi}{2(240)^2}\sqrt{\frac{30\text{E}6 \times 9.61}{0.003225}}$$

$$= 8.154 \text{ Hz}$$

2. Frequency ratio:

$$r = \frac{\bar{f}}{f}$$

$$= \frac{5}{8.154}$$

$$= 0.6132$$

$$\Rightarrow r^2 = 0.376$$

3. Evaluation of dynamic stress.

Using the equidistance h of 12 in.:

$$\text{Eq. } 13 \Rightarrow \sigma_{dyn} = \frac{200(4.5)}{48(12)^2(9.61)(1-0376)}$$

$$\times\left(3(240)^2 108 - 4(108)^3 - 240^3\right)$$

$$= 4{,}352 \text{ psi}$$

4. Dynamic peak deflection:

$$\text{Eq. } 14 \Rightarrow \left(\frac{\delta}{\sigma}\right)_{dyn} =$$

$$\frac{240^3(12)^2}{(30\text{E}6)(4.5)\left(3(240)^2(108) - 4(108)^3 - 240^3\right)}$$

$$= 7.356\text{E}5 \text{ in}^3/\text{lb.}$$

$$\delta_{dyn} = \sigma_{dyn}(7.356\text{E}5)$$

$$= 0.3201 \text{ in.}$$

Computer simulation. A harmonic dynamic analysis of the system has been performed with a widely used pipe stress analysis program. The harmonic load of 200 lb with a frequency excitation of 5 Hz was applied at Node 12 (mid-span). A mode shape analysis was also performed to evaluate the first three natural frequen-

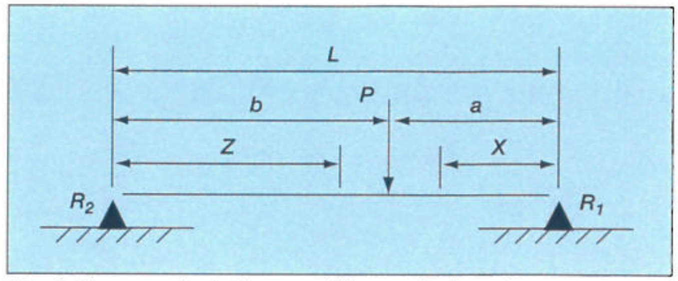

Fig. 4. The same technique could be used where the concentrated load is at any point between the two supports.

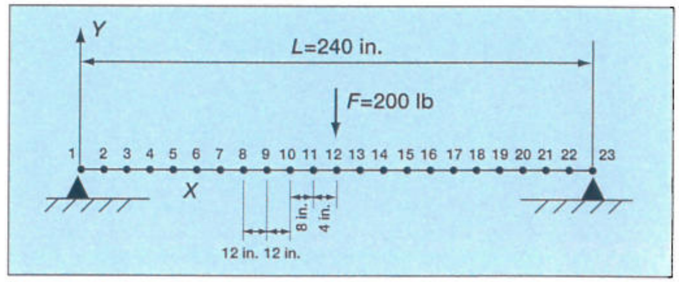

Fig. 5. Simply supported piping model.

Table 1 : Ratio of $\left(\frac{\delta}{\sigma}\right)_{dyn}$ for various loads for the span L = 240 in.

System in consideration				Computer simulation			Eq. 14
Pipe OD	Pipe wall	Force	Frequency	Deflection	Stress	$\left(\frac{\delta}{\sigma}\right)_{dyn}$	
4.5	0.337	200	5	0.3195	4,190	7.62E-5	7.35E-5
4.5	0.74	1,000	5	1.0775	13,948	7.72E-5	7.35E-5
4.5	0.337	400	3	0.4624	6,336	7.3E-5	7.35E-5

cies, which were found to be 8.15, 32.5 and 72.75 Hz. The computer output is listed in Table 1 for comparison. We note that the displacement as well as dynamic stress results are nearly identical in both calculations.

- Stress:

$$\varepsilon = \left|\frac{4352 - 4190}{4352}\right|(100) = 3.72\%$$

- Displacement:

$$\varepsilon = \left|\frac{0.3201 - 0.3195}{0.3201}\right|(100) = 0.18\%$$

LITERATURE CITED

[1] Biggs, J. M., *Introduction to Structural Dynamics,* McGraw-Hill Publishing Company, New York, NY.

[2] *Handbook of Steel Construction,* CISC, Fifth Edition 1991.

***K. T. Truong** has more than 23 years of experience in process piping design and pressure vessel stress analysis using finite element techniques. During this period, he has been lead mechanical engineer and piping consultant on a broad spectrum of refinery, chemical and petrochemical plants for different engineering firms such as ABBDL, SNC/Foster Wheeler, Bantrel, Fluor Daniel, etc. Presently, he is president and co-founder of the Ultragen Group Ltd. Dr. Truong holds BScA, MScA and DScA degrees from Laval University, Quebec. He has served as assistant professor at Moncton University and Université du Québec à Montréal. Dr. Truong is a registered engineer in the province of Quebec and a member of ASME.*