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FINITE ELEMENT METHODS, MODELING, AND NEW APPLICATIONS

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FREE VIBRATION OF A RECTANGULAR PLATE OF NON UNIFORM THICKNESS

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ABSTRACT

The dynamic behavior of a continuous plate on simple supports has been studied using the finite element method. An excellent engineering agreement with some previous techniques is observed in the case of structural elements of uniform thickness. However for vibrating plate of variable thickness, there is a large difference in fundamental frequency obtained in the present work compared to the one based on Rayleigh-Schmidt method.

1. INTRODUCTION

The free vibration problems of a continuous plate of uniform thickness have been discussed by several authors using the transfer matrix, the perturbation, the Laplace transform and the double Fourier series expansion methods (1,2,3,4,5). Recently, Ercoli and Laura (6) determined the fundamental frequency of vibration of a two-span continuous plate using a very simple coordinate functions and the Rayleigh-Schmidt method. The authors conclude by suggesting that the presented method has a considerable advantage for handling elastically restrained support conditions. Unfortunately it has not been possible to assess the relative accuracy of their approach due to the lack of results related to the case of variable thickness.

In the present work a finite element plate model was set up to investigate the dynamic behavior of continuous plates of uniform and nonuniform thickness subjected to the same boundary conditions. The results obtained herein shall be used to verify those reported in the recent literature for the case of transverse vibration of varying thickness plate.

2. FINITE ELEMENT MODEL

Finite element calculations have been carried out for vibration analysis of a two span plate on simple supports. The rectangular flat plate model has its uniform thickness of 1 inch, small in comparison with the short edge of 10 feet.

The homogenuous elastic plate is made of steel having a modulus of elasticity of 30.0E6 psi, a Poisson's ratio of 0.3 and a mass density of 0.733E-3 lb-sec2 /in4.

For the purpose of this analysis, the ANSYS computer code(7), a general purpose commercial finite element software package was utilized. The plate finite element discretization was generated and shown in Fig. 1. A total of 150 elements have been chosen to model the plate in order to optimize the reasonable accuracy of the numerical results versus the computer time. The plate is represented by the quadrilateral shell element (STIF 63) which is defined by four nodal points having six degrees of freedom at each node. The ANSYS node and element generator, PREP7 was used in generating the mesh. The shell element has both bending and membrane capacilities. KEYOPT(1) option was set equal to 2 for neglecting the membrane stiffness to simulate the plate element.

The boundary conditions for simply supported edges are:

$W = 0 \quad M_x = 0 \quad \text{for} \quad X = 0 \quad \text{and} \quad X = a$

$W = 0 \quad M_y = 0 \quad \text{for} \quad Y = 0 \quad \text{and} \quad Y = b$

where W is the displacement normal to XY plane and M the bending moment (Fig. 2)

When the edge is rigidly clamped, all nodal points along this edge shall be restrained for all degrees of freedom.

ANSYS modal analysis (KAN = 2) was performed to determine the natural vibration modes and frequencies. In the case of uniform thickness plate, there are a total of 63 master degrees of freedom specified. The ANSYS program uses a wave-front solution procedure for the system of simultaneous linear equations developed from the assembled finite elements. When the thickness varies, due to the limited in-core wave front size of ANSYS software on University's VAX 11/750 computer version, less than 50 master degrees of freedom are specified. The WAVE step was also called to reorder the element in a specific numbering pattern to minimize the size of the wavefront.

3. RESULTS

The output of modal finite element analysis will give the natural frequencies of the plate which are the property dependant functions. These values were converted into the non-dimensional frequency coefficient using the following equation:

$$\Omega = 2\pi f \sqrt{\frac{\rho h}{D}}\ b^2$$

where
Ω = Modal frequency coefficient (non-dimensional)
f = Natural frequency of plate (cps)
ρ = Mass density lb- sec2/ in4
a = Rectangular edge in X direction, in.
b = Rectangular edge in Y direction, in.
h = Plate uniform thickness , in.
D = Flexural rigidity
$= \frac{Eh^3}{12(1-\nu^2)}$
E = Modulus of elasticity, psi
ν = Poisson's ratio

In the case of plate with nonuniform thickness, the same equation was applied except that h refers to the thickness of the thinner plate, (ho).

For the plate in consideration, with ho = 1 in. and a = 120 in., the frequency coefficient is equal to

$$\Omega = 1.475\ f$$

Before going to the detailed calculation of the dynamic behavior of a two-span continuous plate, verification of ANSYS solution has been performed using a well known problem having closed-form theoretical solution. For a square flat plate with all edges simply supported, Ref(8) reported that the coefficient of the first mode is equal to 19.7 which corresponds to the fundamental frequency (fl) of 13.35 cps for a steel plate having physical dimension data as mentioned above. ANSYS modal solution gave fl equal to 13.02 cps which represents a reasonably accurate comparison with theoretical solution.

Table 1 summarizes the lowest non-dimensional natural frequency coefficient Ω , for various edge conditions in the case of a two-span plate of uniform thickness on simple supports. It also shows the results obtained from previous works to demonstrate the close agreement of finite element solution with other numerical or classical methods.

Tables 2 and 3 illustrate the effect of variable thickness on fundamental frequency coefficients for continuous plate with edges simply supported and clamped respectively. For hl/ho equal to 1.1 where hl is the thickness of the thicker plate, the present results are generally consistent with Ercoli and Laura's work. When the ratio of hl/ho varies from 1.1 to 2.0, there is still a fairly good agreement between two studies for the case of simply supported adges, except for X3 = 0.6 where the divergence is very remarkable (Fig. 3).

However for boundary conditions with two opposite edges rigidly clamped, the first mode vibration frequency calculated by the Rayleigh-Schmidt method is totally different and much higher than these performed by the finite element technique; even though the same trend of variation is generally recognized between two works as long as the thickness varies.

It should be also noted that the results are particularly different when X3 equal to 0.6 (Fig. 4). When observing the results presented by these authors, one can remark that there is practically no difference in the frequency coefficient when the proportion of the thicker plate (X3) varies from 0.3 to 0.6.

In considering vibrations of elastic bodies it is assumed that the system consists of an infinitely large number of particles between which elastic forces are acting. This system requires an infinitely large number of degrees of freedom because any small displacement satisfying the condition of continuity can be taken as a possible or virtual displacement. On this basis, we thought that the fact that a limited degree of freedom was specified will have some influence on the results of vibration analysis. Verification has been performed for both cases of uniform and nonuniform thickness plates with a much smaller number of degree of freedom. The results show however that the fundamental frequency is not really affected for the range of degree of freedom used in this study (less than 1.5%). This allows to conclude that for the case of variable thickness vibrating plate, even if one specifies a higher degree of freedom, the final results shall be unchanged and the natural frequency coefficients obtained by finite element solution are accurate and reliable.

4. SUMMARY and CONCLUSION

1. The finite element solution is consistent with all previous numerical and theoretical solutions for free vibration of continuous flat plate of uniform thickness with various boundary conditions.

2. For plate of non-uniform thickness, there is a fairly good agreement with Rayleigh - Schmidt solution when plate edges are simply supported, except for X3 = 0.6 and hl/ho = 1.5 and 2.0 respectively.

The most serious divergence arises for free vibration of plate with two opposite edges rigidly clamped where the finite element results tend to yield much lower fundamental frequencies.

5. REFERENCES

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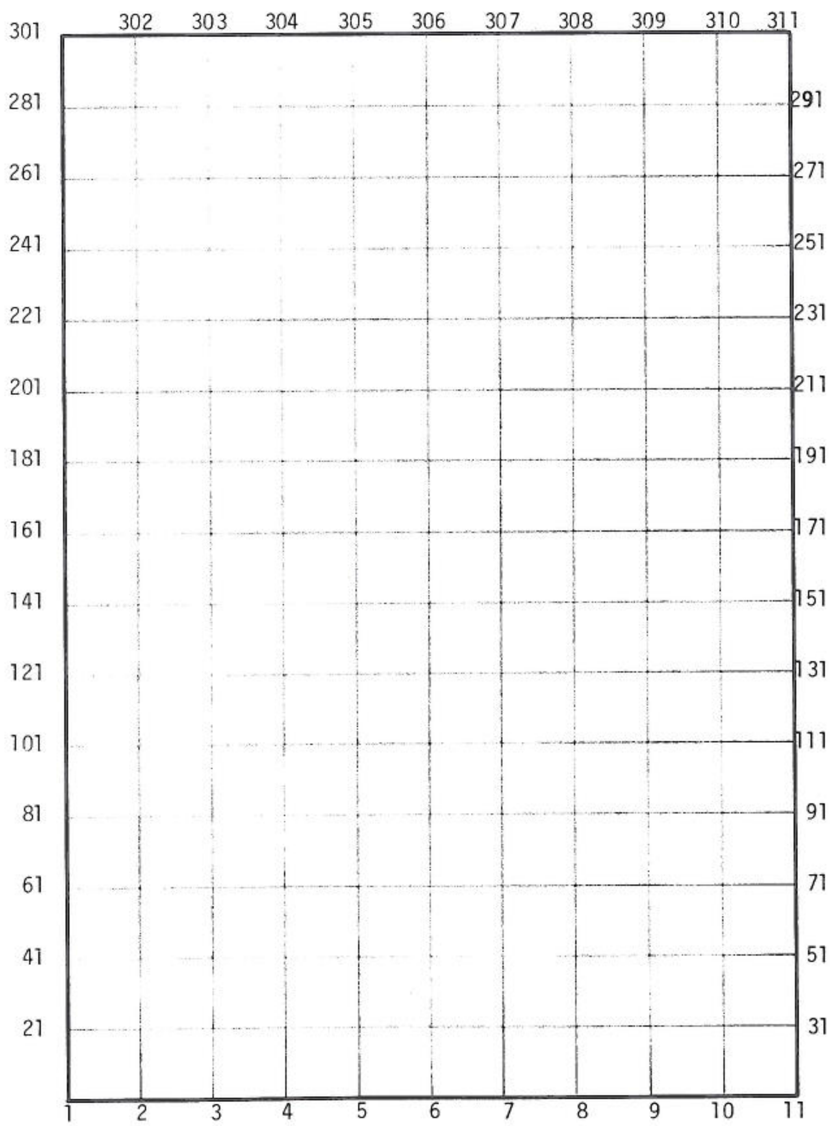


Fig. 1. Plate discretization model

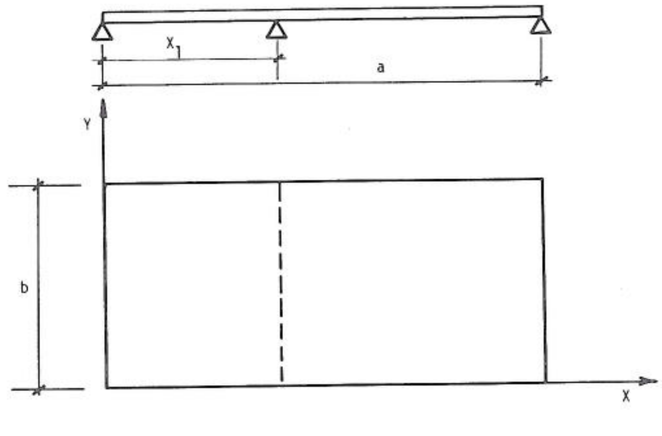


Fig. 2. Continuous plate on simple supports.

Fig. 3. Comparison of Natural Frequency Coefficient of non uniform thickness plate - Simply supported edges and $X_3 = 0.6$

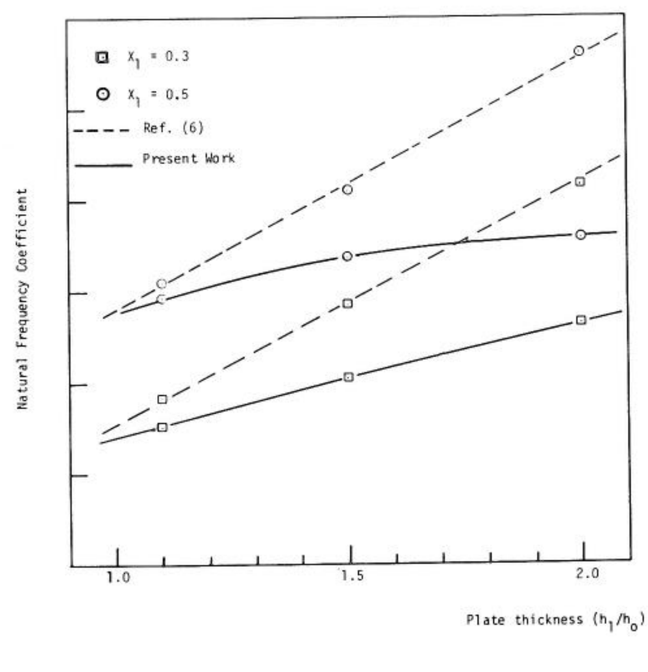


Fig. 4. Comparison of Natural Frequency Coefficient of non uniform thickness Plate - Clamped edges and $X_3 = 0.5$

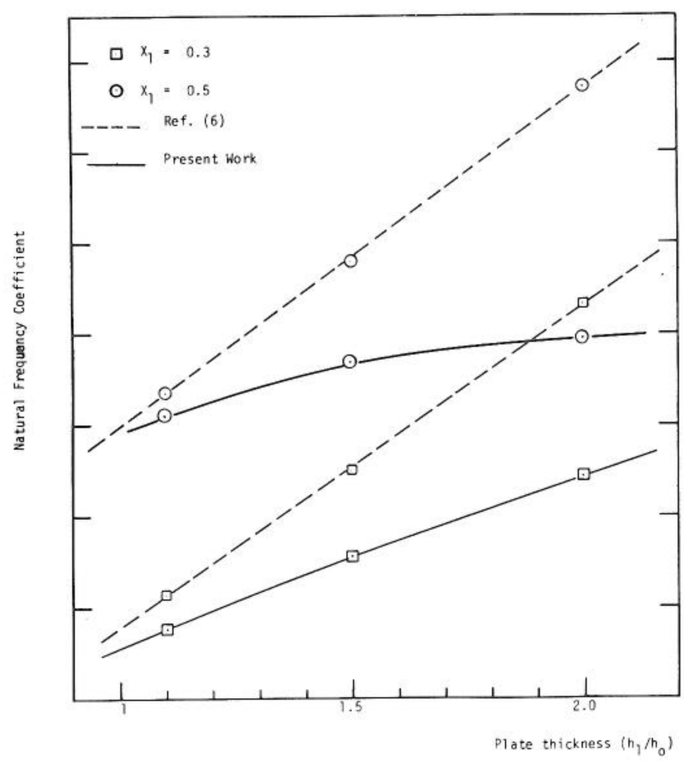
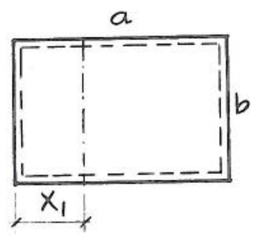
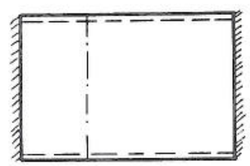
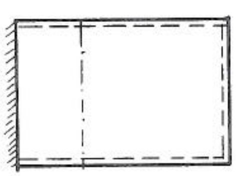
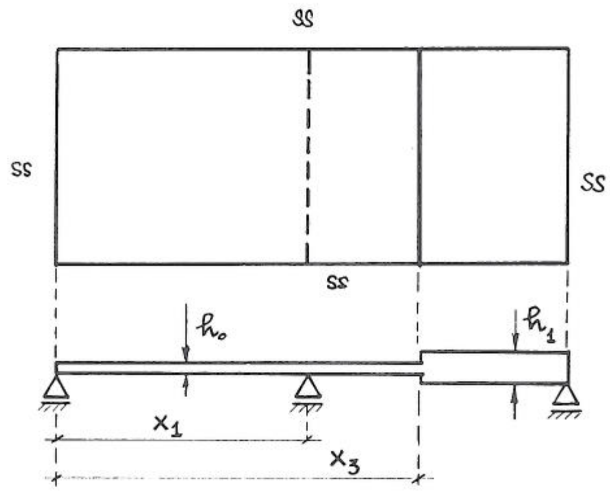


Table 1 Comparison of Natural Frequency Coefficient of Continuous Uniform Thickness Plate

(*) Results obtained in the present work

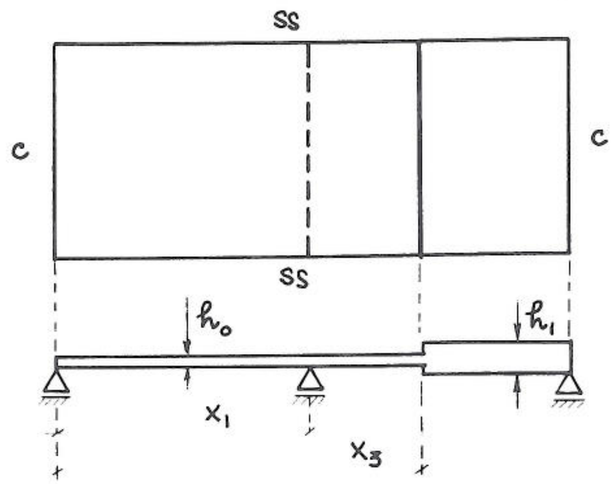
	a/b	X_1 = 0.1	0.2	0.3	0.4	0.5	Ref.
	1	26.392 26.071 25.212	30.193 29.642 28.583	35.804 34.896 33.675	43.606 42.292 41.065	49.625 49.348 47.630	(6) (5) (*)
	2	13.602 13.488 12.870	14.535 14.373 13.701	15.977 15.706 14.986	18.064 17.701 16.919	19.793 19.378 18.841	(6) (5) (*)
	3	11.442 11.385 10.854	11.854 11.272 11.221	12.497 12.355 11.796	13.446 13.246 12.685	14.276 14.256 13.632	(6) (5) (*)
	1	33.350 32.795 32.705	39.288 38.289 37.616	48.023 46.303 45.538	60.341 58.036 57.364	70.172 69.327 68.675	(6) (5) (*)
	2	14.643 14.517 14.088	15.945 15.765 15.219	17.976 17.644 17.107	21.024 20.562 20.044	23.706 23.646 23.114	(6) (5) (*)
	3	11.752 11.681 11.304	12.258 12.181 11.766	13.069 12.951 12.584	14.339 14.158 13.907	15.582 15.579 15.391	(6) (5) (*)
	1	26.773 26.962 -- 25.862	30.756 30.045 30.043 29.146	36.584 35.510 35.508 34.406	45.210 43.664 43.660 42.381	56.127 55.080 55.074 53.610	(6) (5) (4) (*)
	2	13.694 13.528 -- 12.999	14.630 14.430 14.429 13.787	16.059 15.779 15.778 15.081	18.238 17.839 17.839 17.091	21.028 20.813 20.811 20.002	(6) (5) (4) (*)
	3	11.487 11.398 10.919	11.882 11.786 11.251	12.503 12.371 11.826	13.453 13.274 12.737	14.701 14.614 14.099	(6) (5) (*)



		X_1			
		0.3		0.5	
X_3	h_1/h_0	(6)	(*)	(6)	(*)
0.9	1.1	36.012	34.105	49.888	47.949
	1.5	37.236	36.975	51.335	49.782
	2.0	39.609	41.418	53.896	52.023
0.6	1.1	38.297	35.143	52.081	49.309
	1.5	48.454	40.705	62.168	53.836
	2.0	61.644	46.445	76.021	55.863
0.3	1.1	38.485	36.116	52.633	50.651
	1.5	49.416	46.259	63.924	61.571
	2.0	63.434	59.296	78.379	75.492

Table 2 Fundamental Frequency Coefficient of a Continuous Plate of Variable Thickness
(a = b) - Edges simply supported

(*) Results obtained in the present work



		X_1			
		0.3		0.5	
X_3	h_1/h_o	(6)	(*)	(6)	(*)
0.9	1.1	50.151	46.692	72.099	69.786
	1.5	60.637	51.769	80.615	73.683
	2.0	77.383	55.250	92.965	75.620
0.6	1.1	51.345	47.422	73.666	70.930
	1.5	65.080	55.256	87.917	76.902
	2.0	83.220	64.447	107.185	79.458
0.3	1.1	51.617	48.622	74.490	72.725
	1.5	66.206	62.062	90.631	86.356
	2.0	84.242	79.161	110.683	101.479

Table 3 Fundamental Frequency Coefficient of a continuous Plate of Variable Thickness
(a = b) - Two opposite edges clamped

(*) Results obtained in the present work